Wesleyan University, Fall 2025, Quantum Computing, Cryptography, and Networking
Homework 4: Bell Pairs, Teleportation, and QKD
Due by 11:59pm on October 30, 2025

1 Written Problems [10 points]

Problem 1

Recall the Bell basis for two qubits is:

$$\begin{aligned}
|\Phi^+\rangle &= \frac{1}{\sqrt{2}}|0,0\rangle + \frac{1}{\sqrt{2}}|1,1\rangle \\
|\Phi^-\rangle &= \frac{1}{\sqrt{2}}|0,0\rangle - \frac{1}{\sqrt{2}}|1,1\rangle \\
|\psi^+\rangle &= \frac{1}{\sqrt{2}}|0,1\rangle + \frac{1}{\sqrt{2}}|1,0\rangle \\
|\psi^-\rangle &= \frac{1}{\sqrt{2}}|0,1\rangle - \frac{1}{\sqrt{2}}|1,0\rangle
\end{aligned}$$

Let $|\psi\rangle = \sqrt{\frac{3}{4}}|0,0\rangle_{A,B} + \sqrt{\frac{1}{4}}|1,+\rangle_{AB}$. Then, answer the following:

(i) What is the probability of observing the Bell state $|\psi^-\rangle$ if a joint measurement on both qbits is performed? What is the post-measurement state if $|\psi^-\rangle$ is observed?

(ii) What is the probability of observing $|0\rangle$ if a measurement of the A register was made in the Z basis? What is the post-measurement state of both qbits if $|0\rangle$ is observed?

(iii) What is the probability of observing $|0\rangle$ if a measurement of the B register was made in the Z basis? What is the post-measurement state in this case?

Problem 2

Consider the state of two qbits (A,B) as

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{6}}|01\rangle + \frac{i}{\sqrt{6}}|10\rangle + \frac{i}{\sqrt{6}}|11\rangle$$

(i) When A is measured, what is the probability of observing $|0\rangle$? What is the post measurement state?

(ii) When A is measured, what is the probability of observing $|1\rangle$? What is the post measurement state?

Problem 3

Show that $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|-,+\rangle - |+,-\rangle)$.

Problem 4

Use whatever resources you would like (I recommend at least taking a look at Section 6.5 of Quantum Computer Science) to devise an answer to the following question, written in your own words. In teleportation, how is Alice able to send Bob the quantum state $|\psi\rangle$. Recall in teleportation, that first Alice and Bob entangle two qbits. Then alice takes her half of the resulting entanglement and does some operations on both it and $|\psi\rangle$. Those operations result in transforming Bob's half of the original entanglement to $|\psi\rangle$ with Bob only needing two classical bits from Alice to fully recover $|\psi\rangle$.

2 Coding Problems [10 points]

Problem 5

Read about the circuit construction of teleportation given here (see Section 4 at this link):

https://quantum.cloud.ibm.com/learning/en/courses/utility-scale-quantum-computing/teleportation.

You will implement the teleportation circuit using qiskit and submit as teleportation.py. Include the output when your code is run. If you feel up to it, try running your code on IBM's quantum computer rather than the aer simulator. I suggest first sketching out your solution before looking at the final code that is given to correct your own code. Importantly, you will see the use of a u-gate to create the unknown quantum state $|\psi\rangle$.

Problem 6

Implement the quantum key distribution protocol, BB84, that we discussed using qiskit and submit as qkd.py. What fraction of the generated bits are usable in the key? Now add an eavesdropper between Alice and Bob who measures every qbit from Alice before Bob has access. What fraction of the generated bits are usable in the key now?

References

Upload your written work as `hw2.pdf,` and your code solutions as `teleportation.py` and `entanglement.py` to the Google Drive directory I have created for you named `comp411-f25-USERNAME/hw4/`. You should replace `USERNAME` with your Wesleyan username.