Wesleyan University, Fall 2025, Quantum Computing, Cryptography, and Networking
Homework 3: Change of basis, Bell states
Due by 11:59pm on October 9, 2025

1 Written Problems [10 points]

Problem 1

Suppose the measurement basis is $\{|0\rangle, |1\rangle\}$. Compute the probability of observing $|0\rangle$ given the input state $|\psi\rangle = \alpha|+\rangle + \beta|-\rangle$ where $\alpha, \beta \in \mathbb{C}$. Your answer should show that it is clear that the value is a real number.

(Hint: You have done this problem on a previous homework. The only difference here is that $\alpha, \beta \in \mathbb{C}$ instead of $\alpha, \beta \in \mathbb{R}$. You may use the fact that for any $z, w \in \mathbb{C}$, it holds that: $z + z^* = 2\text{Re}(z)$ and $(zw^*)^* = z^*w$ (here z^* is the complex conjugate).)

Problem 2

Let $\mathcal{B}_1 = \{|0\rangle, |1\rangle, |vac\rangle\}$ be an orthonormal basis for $\mathbb{C}^3$. And let $\mathcal{B}_1 = \{|+\rangle, |-\rangle, |vac\rangle\}$ be another orthonormal basis for $\mathbb{C}^3$ defined as $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$. Note that $|vac\rangle$ is the same in both bases but orthogonal to $|0\rangle, |1\rangle, |+\rangle, |-\rangle$.
Given the input state:

$$|\psi\rangle \quad = \quad \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle + \frac{1}{\sqrt{2}}|vac\rangle$$

compute the following.

(i) The probability of observing $|+\rangle$ if measuring in $\mathcal{B}_2$?

(ii) The probability of observing $|vac\rangle$ if measuring in $\mathcal{B}_1$?

(iii) The probability of observing $|0\rangle$ conditioned on a first measurement in basis $\mathcal{B}_2$ yielding a result of $|+\rangle$?

(iv) The probability of observing $|vac\rangle$ conditioned on a first measurement in basis $\mathcal{B}_2$ yielding a result of $|vac\rangle$?

Problem 3

Compute the resulting quantum state after applying the following operations:

$$CNOT(H \otimes H)|+, 0\rangle$$

Write your answer in the Z basis, $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

2 Coding Problems [10 points]

Problem 4

Implement the quantum circuit for the following.

$$CNOT(H \otimes H)|+, 0\rangle$$

Measure in the Z basis. Use the output to verify your answer to Problem 3.

Problem 5

In class, we have constructed a circuit that outputs one of the Bell states (a Bell state is also called an EPR pair, for Einstein, Podolsky, Rosen). In this problem, you will use qiskit to construct this circuit as well as circuits that output the other three Bell states. The Bell states are prime examples of entangled states.

You will construct four circuits using qiskit, for $|\phi^+\rangle$, $|\phi^-\rangle$, $|\psi^+\rangle$, and $|\psi^-\rangle$, respectively. For each circuit, briefly describe your thoughts in designing the circuit. Run each circuit and convince yourself that your design indeed provides the desired output.

References

Upload your written work as `hw3.pdf`, and your code solutions as `cnot.py`, `bell.py`, to the Google Drive directory I have created for you named `comp411-f25-USERNAME/hw3/`. You should replace `USERNAME` with your Wesleyan username.