Wesleyan University, Fall 2025, Quantum Computing, Cryptography, and Networking
Homework 2: Measuring Qbits
Due by 11:59pm on Thursday, October 2, 2025

1 Written Problems [15 points]

Problem 1

Let $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, let $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and let

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \tag{1}$$

$$|-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} \tag{2}$$

Show that $\{|+\rangle, |-\rangle\}$ is a basis of $\mathbb{C}^2$ using the following two methods.

(i) In Method 1, we show that the above holds directly using the definition of basis. That is, we show that $\forall |\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \in \mathbb{C}^2$, where $\alpha, \beta \in \mathbb{C}$, $|\psi\rangle$ can be represented as a linear combination of $\{|+\rangle, |-\rangle\}$.

(ii) In Method 2, we leverage the fact that $\{|0\rangle, |1\rangle\}$ is a basis of $\mathbb{C}^2$, and show that $\{|0\rangle, |1\rangle\}$ can be represented as a linear combination of $\{|+\rangle, |-\rangle\}$. Therefore, $\{|+\rangle, |-\rangle\}$ is also a basis of $\mathbb{C}^2$.

Hint: you can first show that

$$|0\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}} \tag{3}$$

$$|1\rangle = \frac{|+\rangle - |-\rangle}{\sqrt{2})} \tag{4}$$

This is a useful result that you will want to keep in mind. For this homework, you need to derive this result yourself. After this homework, you can use them as a known result. Note that you can derive them directly from the definition of $|+\rangle$ and $|-\rangle$.

Problem 2

Suppose the measurement basis is $\{|0\rangle, |1\rangle\}$. What is the probability of observing $|0\rangle$ given the input state $|\psi\rangle = \alpha|+\rangle + \beta|-\rangle$? For simplicity, suppose that $\alpha, \beta \in \mathbb{R}$. Obtain the results using the following two methods.

(i) In Method 1, we first use Eq. (1) and (2) to represent $|\psi\rangle$ in $|0\rangle$ and $|1\rangle$. Then we can use Born's rule to obtain the probability of observing $|0\rangle$.

(ii) In Method 2, we use the inner product to directly obtain the probability of observing

$|0\rangle$.

Problem 3

Let $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. For simplicity, suppose that $\alpha, \beta \in \mathbb{R}$. Suppose we measure $|\psi\rangle$ in X basis, i.e., $\mathcal{B} = \{|+\rangle, |-\rangle\}$. What is the probability of observing $|+\rangle$? You only need to use the inner product to obtain the result.

2 Coding Problems [5 points]

Problem 4

In homework 1, we created some simple circuits and measured them. In this homework, we will go one step further and run the circuit. To run the circuit, we need some hardware on which to run it. While IBM allows us to run our code on a small quantum computer (i.e., real quantum hardware) for free, this is a time-limited resource. So for now, we will run our code on a simulation of a quantum computer rather than the real thing. To do this, we will use aer_simulator.

Take your entanglement.py code from hw1 and add the following line to the top of your code.

```
from qiskit\_aer import Aer
```

Then add the following block of code to the bottom of your entanglement.py.

```
sim = Aer.get_backend('aer_simulator')
result = sim.run(circuit).result()
counts = result.get_counts()
print(counts)
```

Now run your code and include your output in your homework submission. You will see an output result printed, something like the following.

```
{'11': 519, '00': 505}
```

What does this mean? The Aer simulator ran the code 519+505=1024 times (aka 1024 shots). For 519/1024 times, the state $|11\rangle$ was the result. The other 505/1024 times the state $|00\rangle$ was the result. The more shots the closer our experimental distribution will get to what we expect to observe theoretically. You can change the number of shots with the following.

```
simulator.run(circuit, shots=1024)
```

Problem 5

Take your swap.py code from hw1. Add the lines from the previous problem to use the aer_simulator. Change the number of qbits in the circuit from 2 to 20 and run the circuit. What do you observe? Does the result make sense to you? Include your output and your explanation in your homework submission.

Problem 6

Create a circuit with 20 qbits. Apply a Hadamard to each qbit. Then run the circuit. Note that you can use the following to measure all qbits at once.

```
circuit.measure_all()
```

What is the output (you can just include a snippet)? Does the result make sense to you?

References

Upload your written work as hw2.pdf, and your code solutions as `entanglement.py`, `swap.py`, and `hadamard.py`, to the Google Drive directory I have created for you named `comp411-f25-USERNAME/hww/`. You should replace `USERNAME` with your Wesleyan username.