Wesleyan University, Fall 2025, Quantum Computing, Cryptography, and Networking
Homework 1: From classical bits to quantum bits
Due by 11:59pm on Thursday, September 25, 2025

1 Written Problems [15 points]

Problem 1

Explain whether the following is a valid qbit state and why: $\frac{1-i}{2\sqrt{2}}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$.

Problem 2

Consider the following single qbit states.

$$\begin{aligned} |+\rangle &= \frac{|0\rangle + |1\rangle}{\sqrt{2}} \\ |-\rangle &= \frac{|0\rangle - |1\rangle}{\sqrt{2}} \end{aligned}$$

Verify that these are unit vectors.

Problem 3

If $|\alpha\rangle = i|0\rangle + 7|1\rangle$ and $|\beta\rangle = 3|0\rangle + |1\rangle$, compute $\langle\alpha|\beta\rangle$.

Problem 4

Consider the following gate operation: regardless of the input, it always produces a constant output. Is this a valid quantum gate? Justify your answer.

$$\begin{aligned} \text{Gate}|0\rangle &= |0\rangle \\ \text{Gate}|1\rangle &= |0\rangle \end{aligned}$$

Problem 5

Consider a gate U that transforms the Z-basis states, $|0\rangle, |1\rangle$ as follows.

$$\begin{aligned} U|0\rangle &= \frac{\sqrt{3}}{2}|0\rangle + \frac{\sqrt{3}+i}{4}|1\rangle \\ U|1\rangle &= \frac{\sqrt{3}+i}{4}|0\rangle - \frac{\sqrt{3}+3i}{4}|1\rangle \end{aligned}$$

Suppose that $\psi = \alpha|0\rangle + \beta|1\rangle$ is a normalized quantum state, i.e., $|\alpha|^2 + |\beta|^2 = 1$.

(i) Calculate $U|\psi\rangle$

(ii) Given your answer in (i), is U a valid quantum gate? Justify your answer.

2 Coding Problems [5 points]

In this problem, you will try out IBM's Quantum Composer to get some intuition about constructing and visualizing quantum circuits. The IBM Quantum Composer tool can be found at

`https://www.ibm.com/quantum-computing/tools/`

Documentation can be found here:

`https://quantum.cloud.ibm.com/docs/en/guides/composer`

Note that the order of qbits is read bottom to top. The bottom qbit will be the leftmost one in the ket.

Problem 6

In this problem, you will try out IBM's Quantum Composer to get some intuition about constructing and visualizing quantum circuits. The IBM Quantum Composer tool can be found at https://www.ibm.com/quantum-computing/tools/. Documentation can be found here: https://quantum.cloud.ibm.com/docs/en/guides/composer. Note that the order of qbits is read bottom to top. The bottom qbit will be the leftmost one in the ket.

(i) We first build a SWAP circuit out of CNOTs.

- Name your circuit "swap" in the top left.
- Switch from OpenQASM 2.0 code in the top right to Qiskit code.
- Create a circuit with two qbits and two classical bits. The number of quantum and classical registers can be set under Edit, then Manage Registers.
- By default, quantum registers are initialized to be in state $|0\rangle$. We can change the state in different ways depending on the gate applied. To change the state of the top qbit from $|0\rangle$ to $l1\rangle$ add a Pauli-X gate, aka NOT gate. The state of both qbits at this point should be $01\rangle$.
- Add 3 CNOT gates. For the first and third CNOT gates the top qbit will be the control bit. for the second CNOT gate, the bottom qbit will be the control bit. Your state at this point should be $10\rangle$. Under View you can select Panels and then State vector to display the quantum state vector.
- Measure your qbits. For each qbit, add a measurement output in one of the classical bits. What are the probabilities for the different computational basis states after adding the first measurement on the top qbit? What are the probabilities for the different computational basis states after adding the second measurement on the bottom qbit?
- Take a screenshot of your circuit, probabilities, state vector and code and include it in your write-up.
- Copy your qiskit code and save it as swap.py and submit it separately with your write-up.

(ii) We next build a circuit that entangles two qbits.

- Name your circuit "entanglement" in the top left.
- Create a circuit with two qbits and two classical bits.
- Add a Hadamard gate to the top qbit. How did the statevector change?
- Now add a CNOT gate with the top qbit as the control bit. What is the statevector now?
- Measure your qbits. For each qbit, add a measurement output in one of the classical bits. What are the probabilities for the different computational basis states after adding the first measurement on the top qbit? What are the probabilities for the different computational basis states after adding the second measurement on the bottom qbit?
- Take a screenshot of your circuit, probabilities, state vector and code and include it in your write-up.
- Copy your qiskit code and save it as entanglement.py and submit it separately with your write-up.

Problem 7

Over the course of the semester, we will use IBM's Qiskit python library. The goal of this problem is to have you get qiskit installed locally for you.

(i) Follow the installation instructions here to install qiskit:

`https://quantum.cloud.ibm.com/docs/en/guides/install-qiskit`

(ii) Take the entanglement.py file that you saved in the previous problem. Add the line "print(circuit.draw())" at the end and run your code. You should see a drawing of the circuit you created pop up. Include a screenshot of this circuit in your write-up.

3 Submission

Upload your written work as `hw1.pdf`, and your code solution as `hw1.py`, to the Google Drive directory I have created for you named `comp411-f25-USERNAME/hw1/`. You should replace `USERNAME` with your Wesleyan username.