1. WRITTEN PROBLEMS (5 POINTS)

PROBLEM 1. In Problem 2 you will implement a function to perform a linearity check on a linkedlist: if the linked-list is linear, the function returns true, otherwise the function returns false.

In this problem, you will analyze the cost of the linearity check you implemented. If the number of nodes in the linked list is n, give the cost of your linearity check algorithm in terms of n and explain how you computed that cost. You must include your code for Problem 2 in your write-up as well as in hw8.c. This will greatly facilitate our checking of your cost computation.

Solution: One approach you may have taken in Problem 2 is to loop through the linked list, and for each node, n, loop through the linked list again (until the node just prior to node n), comparing the address of each node to that of node n. If the addresses ever match, you have a loop. In this case, you have an inner loop that has a constant-cost body and iterates up to n times (where n is the number of nodes), for a total cost of O(n). The cost of the body of the outer loop is the cost of executing the inner loop, so O(n), and the outer loop is also iterated up to n times, so the total cost is $O(n^2)$.

Alternatively, you might have implemented a more efficient approach where you loop through the linked list keeping track of two pointers into the list, advancing one pointer one node at a time and the other pointer two nodes at a time. If the two pointers are ever equal, or if the faster pointer's next node address ever equals the slower pointer then there is a loop. If either pointer ever reaches the end of the list, then the list is linear. Because we only loop through the linked list once (albeit doing a larger constant time amount of work), the cost is O(n). To see the intuition for this, consider the following. First, ignore the time spent for the tortoise to enter the loop (which is at most n). We'll instead consider the situation when the tortoise enters the loop, call this node 0. Then let x be location of the hare in the loop when the tortoise enters the loop: specifically xis how many nodes the hare is behind the tortoise. After t iterations (i.e., t nodes are traversed), the tortoise will be at location $t \mod \lambda$ and the hare will be at location $(2t - x) \mod \lambda$. How many iterations, t, will it take before the location of the hare equals that of the tortoise? Until $t \mod \lambda = (2t - x) \mod \lambda$. Solving for t we get t = x, which means after x iterations, the hare and the tortoise will meet. Since x is upper-bounded by λ and λ is upper-bounded by n, then the cost is O(n).

2. Programming problems (15 points)

PROBLEM 2. In this problem you will implement a linearity test to check that a linked-list is linear (that is, traversing next fields results no loops and eventually gets to a NULL node). Such a test is important for checking the correctness of functions: any operations performed on a linked-list should maintain the linearity of the linked-list.

Your function will take the head of a linked-list as input, and return true if the linked-list is linear and return false if the linked-list is non-linear. By non-linear, we mean that as you loop through the linked-list, eventually one of the nodes visited is a node that has already been visited before. The linearity function has been specified for you in hw8.h. I have already specified a linked-list node type for you (struct qnode) in hw8.h.

Hint 1: You can compare two variables of type pointer to check for equality. If the values of the pointers (i.e. addresses contained in them) are the same, then the pointers really do contain the same memory address and the == operator will return true.

Hint 2: A straightforward algorithm to check linearity compares each node to all of the nodes preceding it. There is also a less straightforward but more efficient algorithm in which two pointers both traverse the list, one pointer traversing the list twice as fast as the other pointer. If either pointer ever reaches the end of the list then the list is linear. If the two pointers ever end up on the same node, then the list is not linear.

PROBLEM 3. Use a linked-list to implement a queue (an ordinary queue, not a priority queue). In a bit more detail, you will implement the following:

- qnode structure: an individual element in the queue. Note that this struct has already been implemented for you.
- queue structure: a structure with a qnode field that represents the linked-list that represents the queue, along with any other fields you need.
- create function: initialize a new queue.
- is_empty function: check whether the queue is empty.
- enqueue function: add an element to the back of queue. Your function should take constant time.
- dequeue function: remove an element from the front of the queue. Your function should take constant time.
- as_array function: fill an array with the contents of the queue.
- **size** function: return the size of the queue. For full credit, your function should be constant time, although a linear-time solution will receive some credit.
- print function: print all of the elements in the queue.

The functions are specified in hw8.h, and you will implement them in hw8.c. Using the function you wrote in Problem 2, add asserts to your code to check for linearity before your enqueue and dequeue functions return. The header file hw8.h has an empty definition of a queue structure; it is up to you to fill in the details. Do not modify any other part of hw8.h.

3. Code distribution

As usual, the code distribution contains a driver program (make driver) and a tests program (make tests). The driver program is used to build a queue using your implementation. Make sure

<pre>\$./driver (0) Exit (1) Create queue</pre>	(0) Exit (1) Create queue (2) Enqueue character
(2) Enqueue character	(3) Dequeue character
(3) Dequeue character	(4) Print queue
(4) Print queue	Enter choice: 2
Enter choice: 1	Enter character: 3
Queue contents:	Queue contents:
(0) Exit	Q: abc3
(1) Create queue	(0) Exit
(2) Enqueue character	(1) Create queue
(3) Dequeue character	(2) Enqueue character
(4) Print queue	(3) Dequeue character
Enter choice: 2	(4) Print queue
Enter character: a	Enter choice: 3
Queue contents:	Queue contents:
Q: a	Q: b c 3
(0) Exit	(0) Exit
(1) Create queue	(1) Create queue
(2) Enqueue character	(2) Enqueue character
(3) Dequeue character	(3) Dequeue character
(4) Print queue	(4) Print queue
Enter choice: 2	Enter choice: 3
Enter character: b	Queue contents:
Queue contents:	Q: c 3
Q: a b	(0) Exit
(0) Exit	Create queue
(1) Create queue	(2) Enqueue character
(2) Enqueue character	(3) Dequeue character
(3) Dequeue character	(4) Print queue
(4) Print queue	Enter choice: 2
Enter choice: 2	Enter character: g
Enter character: c	Queue contents:
Queue contents:	Q: c 3 g
Q: a b c	-

FIGURE 1. A sample session using the linked-list-backed queue driver program.

to test your is_linear function on lists that are linear and on lists that are not! Figure 1 shows a sample session using the driver.

4. SUBMISSION

Submit your written work as hw8.pdf and your code as hw8.c and hw8.h to the Google Drive directory I have created for you named comp211-f23-USERNAME/hw8/. You should replace USERNAME with your Wesleyan username.

Do not forget that your written work must be submitted as a PDF! And make sure that at the top of each file you have put your name! Do not, however, change the names of the files.